$$f'(x) = \frac{dy}{dx}$$ when $y = f(x)$

another 

suggests $\frac{\Delta y}{\Delta x} = m$

$\frac{dy}{dx}$ suggests $\frac{\Delta y}{\Delta x}$ when you are looking at infinitesimally small differences

why "d" and not "Δ"

$\Rightarrow$ d is meant to suggest

$$\lim_{\text{as } \Delta \to 0} \Delta = d$$

In fact dy and dx are called DIFFERENTIALS

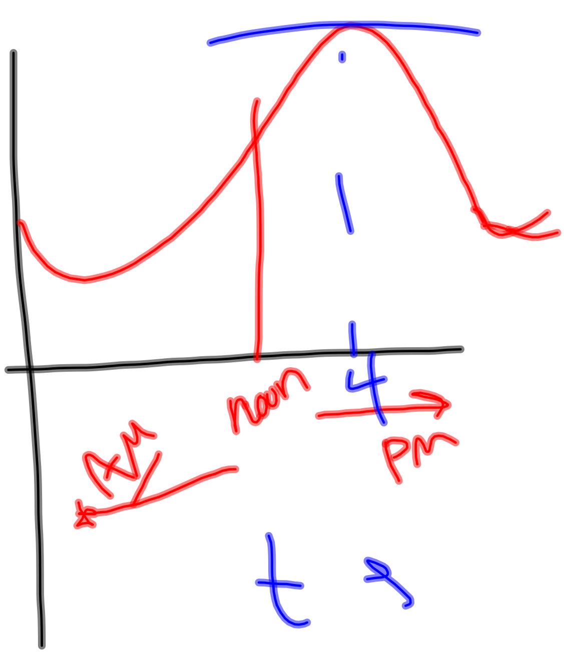
I.e. IT IS NOTATION

THERE IS NOTHING to UNDERSTAND (yet)

3.1/15)

b) ~~rts~~ rate at which temp rises

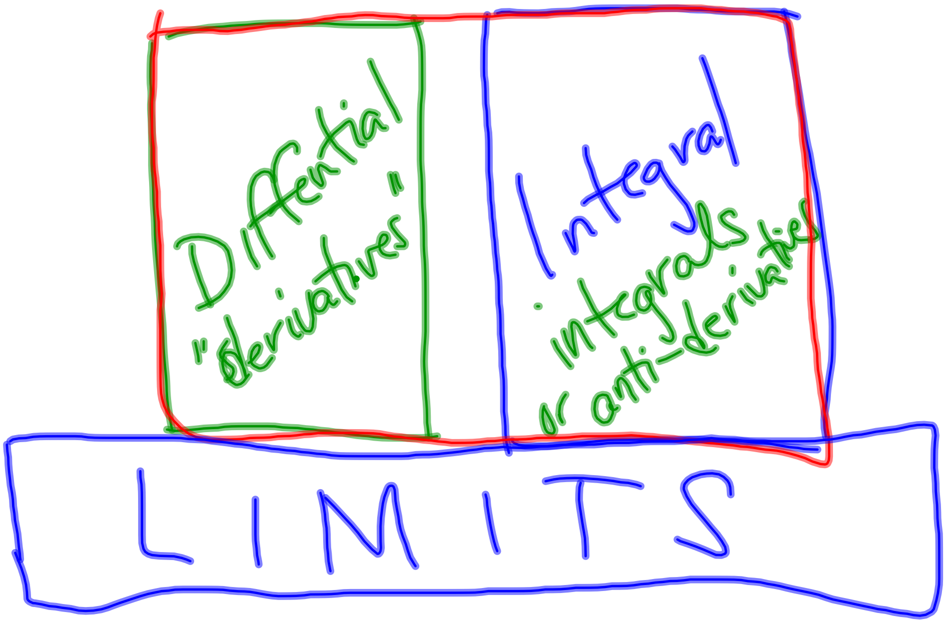
$$= \frac{\text{total rise}}{\text{total time}}$$



$$= \frac{\Delta °F}{\Delta t} \approx 5°/h$$

c) decrease most rapidly

CALCULUS


Differential
"derivatives"
Integral
integrals
or anti-derivatives
LIMITS

3.1/18) rock falls 576 ft

$d = 16t^2$

a) $16t^2 = 576 \cancel{ft}$

$t^2 = 36 \Rightarrow t = \pm 6$

physical situation throw out -6

6 sec

(b) ave vel. m. line joining 2 pts

pt 1: d: +576 ft
t: +0 sec

pt2: d: 0 ft
t: 6 sec

$m = \frac{576-0}{0-6} = -96$ ft/sec

c) avg vel$_{3\,sec}$: pt 1: 576 ft / t = 0

pt 2: $16(3)^2$ ft / t = 3

$$\text{avg vel} = \frac{576-(576-16(3)^2)}{-3}$$

$$= \frac{576-(576-144)}{-3} = \frac{144}{-3} = -48 \text{ ft/sec}$$

($16t^2$ isn't height, it's how far I fall)

d) $$V_{inst} = \lim_{t_1 \to 6} \frac{(576-16t_1^2)-0}{t'-6}$$

$$= \lim_{t_1 \to 6} \frac{576-16t_1^2}{t'-6} = \lim_{t_1 \to 6} \frac{16(36-t_1^2)}{t_1-6}$$

$$= \lim_{t_1 \to 6} \frac{16(6-t_1)(6+t_1)}{-(6-t_1)}$$

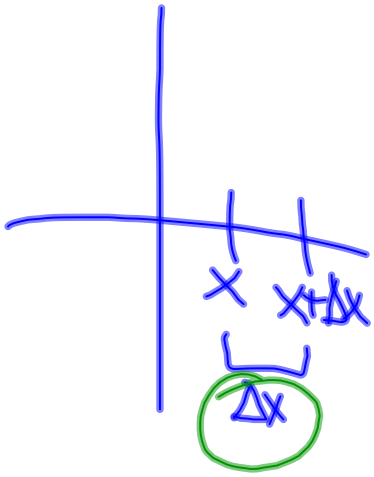
$$= \lim -16(6+t_1) = -192 \text{ ft/s}$$

Q? $\dfrac{\left(\dfrac{7}{8}\right)}{9} = \dfrac{7}{8 \cdot 9}$? $\neq \dfrac{7}{\left(\dfrac{8}{9}\right)}$

$$\frac{\left(\frac{7}{8}\right)}{9} \qquad 7 \div 8 \div 9$$

3.2/15) find $\frac{dy}{dx}$ if $f(x) = \frac{1}{x}$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - (x)}$$



$$= \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{(x)(x+\Delta x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{(\Delta x)(x)(x+\Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = \frac{-1}{x(x)}$$

"$x(x+0)$"

$$= -\frac{1}{x^2}$$

Note: $(x+\Delta x)-x = \Delta x$

BUT

$f(x+\Delta x)-f(x)$ is NOT

SIMPLIFY-able

OLD way:

$$\frac{dy}{dx} = f'(x) = \lim_{w\to x} \frac{f(w)-f(x)}{w-x}$$

$$= \lim_{w\to x} \frac{\frac{1}{w}-\frac{1}{x}}{w-x} = \lim_{w\to x} \frac{\frac{x-w}{xw}}{w-x}$$

$$= \lim_{w\to x} \frac{(x-w)}{xw(w-x)} = \lim_{w\to x} \frac{-1}{xw}$$

$$= \frac{-1}{x\cdot x} = \frac{-1}{x^2}$$

"I understand $\lim\limits_{w \to x} \dfrac{f(w)-f(x)}{w-x}$ but I don't understand $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x \to 0}$"

But $w = x + \Delta x$

I hear: I am comfortable using $\lim\limits_{w \to x}$ but not $\lim\limits_{\Delta x \to 0}$

coming

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

3.2/16) $f(x) = \frac{1}{x+1}$

$$f(x+\Delta x) = \frac{1}{(x+\Delta x)+1}$$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+1} - \frac{1}{x+1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{(x+1)-(x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+1)-(x+\Delta x+1)}{(\Delta x)(x+1)(x+\Delta x+1)}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(\Delta x)(x+1)(x+\Delta x+1)} = \lim_{\Delta x \to 0} \frac{-1}{(x+1)(x+\Delta x+1)}$$

$$= \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}$$

lim w/ specific # gives me slope at a pt.

lim w/ letter/symbol gives me a FUNCTION that gives me slope at ANY pt.

PATTERN

Power Rule

when $y = x^n$,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) = nx^{(n-1)}$$

$$\frac{d}{dx}\left(x^1\right) = 1x^0 = 1$$

$$\frac{d}{dx}\left(x^2\right) = 2x^{2-1} = 2x$$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\frac{d}{dx}\left(x^4\right) = 4x^3$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = (-1)x^{-2} = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$\frac{d}{dx}\left(3x^2-2x+\frac{2}{x}\right)?$$

$$=\frac{d}{dx}\left(3x^2\right)-\frac{d}{dx}(2x)+\frac{d}{dx}\left(\frac{2}{x}\right)$$

$$=3\frac{d}{dx}\left(x^2\right)-2\frac{d}{dx}(x)+2\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$=3\frac{d}{dx}\left(x^2\right)-2\frac{d}{dx}(x)+2\frac{d}{dx}\left(x^{-1}\right)$$

$$=3(2x)-2(1)+2\left(\frac{-1}{x^2}\right)$$

3.2/20 $\quad y = \frac{1}{x^2} \quad f(x) = \frac{1}{(\ x\ )^2} \quad f(x+\Delta x) = \frac{1}{(x+\Delta x)^2}$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - (x)}$$

def^n #3

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}}{\Delta x} =$$

$\frac{\frac{a}{b}}{c} = \frac{a}{bc}$

$$= \lim_{\Delta x \to 0} \frac{\frac{x^2 - (x+\Delta x)^2}{x^2 (x+\Delta x)^2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 - (x^2 + 2x\Delta x + (\Delta x)^2)}{(\Delta x) x^2 (x+\Delta x)^2}$$

$$= \lim_{\Delta x \to 0} \frac{-2x\Delta x - (\Delta x)^2}{(\Delta x) x^2 (x+\Delta x)^2} = \lim_{\Delta x \to 0} \frac{\Delta x(-2x - \Delta x)}{\Delta x(x^2 (x+\Delta x)^2)}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{x^2 (x+\Delta x)^2} = \frac{-2x - 0}{x^2 (x+0)^2} = \frac{-2}{x^3}$$

3.2/20

"old way"

$$f'(x) = \lim_{w \to x} \frac{\frac{1}{w^2} - \frac{1}{x^2}}{w - x}$$

$$= \lim_{w \to x} \frac{x^2 - w^2}{(w-x)w^2x^2}$$

$$= \lim_{w \to x} \frac{(x-w)(x+w)}{(w-x)w^2x^2} = \lim_{w \to x} \frac{-(x+w)}{w^2x^2}$$

$$= \frac{-2x}{x^4} = \frac{-2}{x^3}$$

3.2/18

$y = x^2 - x$ $\quad f(x) = x^2 - x$

$f(x+\Delta x) = (x+\Delta x)^2 - (x+\Delta x)$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - (x+\Delta x) - (x^2 - x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x - \Delta x - x^2 + x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 1)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 1)$$

$$= 2x + 0 - 1 = 2x - 1$$

Advanced f^n notation

$f(x) = x^2 - x$

$f(2) = 2^2 - 2$

$f(-7) = (-7)^2 - (-7)$

$f(\ ) = (\ )^2 - (\ )$

$f(2) = (2)^2 - (2)$

$f(e^x) = (e^x)^2 - (e^x)$

$f(x+\Delta x) = (x+\Delta x)^2 - (x+\Delta x)$

32

17) $y = ax^2 + b$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 + b - (ax^2+b)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x^2 + 2x\Delta x + (\Delta x)^2) + b - ax^2 - b}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ax^2 + 2ax\Delta x + a(\Delta x)^2 + b - ax^2 - b}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2ax\Delta x + a(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2ax + a\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} 2ax + a\Delta x$$

$$= 2ax$$

Big Fat Cheaters +N.S.

Compare problems 3.2/15 and 16

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2} \qquad \frac{d}{dx}\left(\frac{1}{x+1}\right) = \frac{-1}{(x+1)^2}$$

lim w/specific # gives me slope at a pt.

lim w/ letter/symbol gives me a FUNCTION that gives me slope at ANY pt.

Mega Pattern

Power Rule

$$\frac{d}{dx}\left(x^n\right) = nx^{(n-1)}$$

$y = x^n$

$\frac{dy}{dx}$

$$\frac{d}{dx}(x^1) = (1)x^{1-1}$$
$$= 1x^0 = 1$$

$$\frac{d}{dx}(x^2) = 2x^1$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(x^4) = 4x^3$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right)$$
$$= (-1)x^{-2} = \frac{-1}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}\left(x^{-2}\right) =$$
$$(-2)x^{-3} = \frac{-2}{x^3}$$

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$$
$$= \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-3/2}$$
$$= -\frac{1}{2x^{3/2}}$$

$$y = 3x^2 - 14x + \frac{2}{x} - \frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(3x^2 - 14x + \frac{2}{x} - \frac{1}{\sqrt{x}}\right)$$

$$= \frac{d}{dx}\left(3x^2\right) - \frac{d}{dx}\left(14x\right) + \frac{d}{dx}\left(\frac{2}{x}\right)$$

$$- \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$= 3\frac{d}{dx}\left(x^2\right) - 14\frac{d}{dx}\left(x\right) + 2\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$- \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

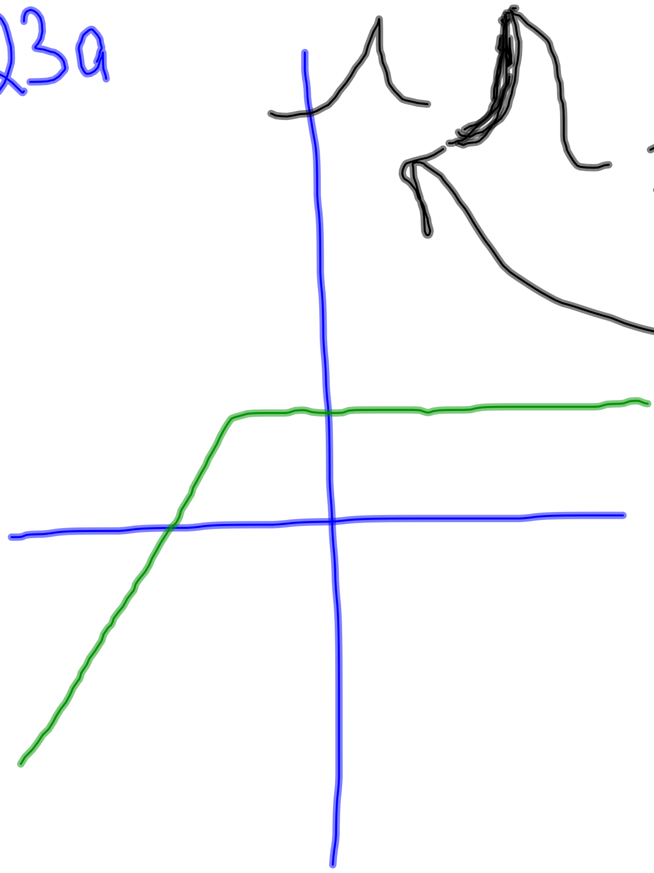
$$= 3(2x) - 14(1) + 2\left(\frac{-1}{x^2}\right)$$

$$+ \frac{1}{2x^{3/2}}$$

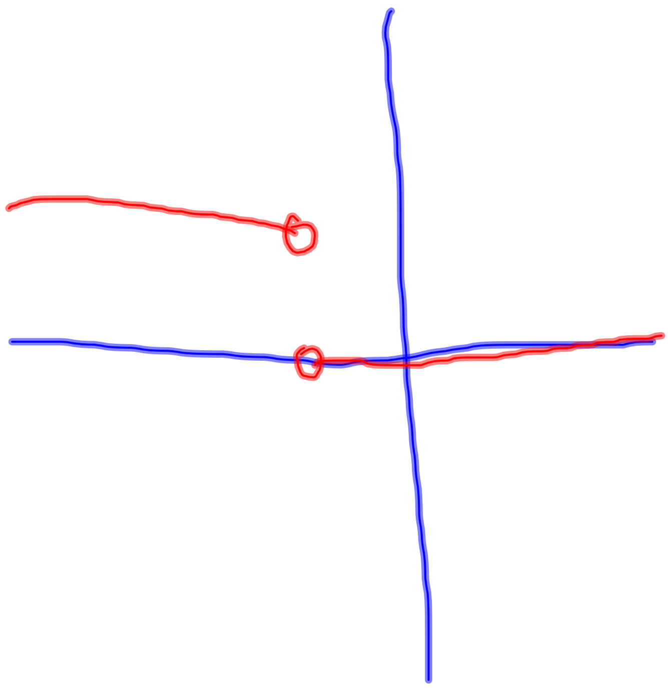
$$\frac{d}{dx}(2x) = 2\left(\frac{d}{dx}(x)\right)$$

$$\frac{d}{dx}\left(\frac{1}{2}x\right) = \frac{1}{2}\left(\frac{d}{dx}(x)\right)$$

3.2/23a



fns with corners [or cusps] have no derivative



23b